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Examining Volatility Contagion in the Crude Oil Market

[1]Justin Odadami EJUKWA, and [2] Godwin Lebari TUANEH.
[1]Department of Mathematics & Statistics,
Ignatius Ajuru University of Education Port Harcourt, Nigeria.
[2]Department of Agriculture & Applied Economics,
Rivers State University, Port Harcourt, Nigeria.
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Abstract
Adopting four primary crude oil benchmarks and using data spanning from 1982 to 2022 sourced from central Bank of Nigeria (CBN) statistical bulletin, this research used the Multivariate GARCH, to analyze the effects of the returns, and volatility spillovers and also assessed how the association between benchmark crude oil prices evolves and holds up across time. Since crude oil benchmark factors such as mean, time-varying covariance, and spillover volatility are interdependent, it was consequently necessary to use a multivariate GARCH Model to assess the advantages of this dependency. Particularly, the diagonal BEKK model was deployed along with the constant conditional correlation. The findings demonstrated from the diagonal BEKK model that historical conditional volatility and squared errors had substantial impact on the conditional variances of the four mean returns for crude oil benchmarks. The result from the conditional covariances demonstrated significant effects of cross products of earlier error terms and preceding covariance terms. The research validated the substantial volatility co-movements and spillover across crude oil markets. The sufficiency tests showed that the model was adequate. the study concluded that the volatility of the crude oil markets exhibited strong linkages and bilateral volatility transmission from one market to the other. Also, the constant conditional correlation of the DCC-GARCH showed that there was no disparity between correlation of the expected returns of the Average, Brent, Dubai, and West Texas intermediate raw price and it's return. The portmanteau test and the QQ plot test showed that the diagonal BEKK-GARCH model was sufficient. The study recommended that the central bank should prioritize price stabilization to avoid petro-aggression. Also, the government should enhance its system for recognizing volatility spillover effects between Average, Brent, Dubai, and WTI Crude Oil returns enhance the early warning system for crude oil price crises and to reduce price volatility.

Keywords: *Crude oil Market, Constant Conditional Correlation Diagonal BEKK, Multivariate GARCH Model, Volatility Contagion*

1 Background to the Study

Crude oil is one of the most widely traded commodities in the world, playing a crucial role in the global economy. The price of crude oil is known to exhibit significant fluctuations over time resulting to shocks. Volatility spillover refers to the transmission of volatility shocks from one market to the other, while time-varying volatility captures the changing nature of volatility over time. To find the changing correlations and volatility transfer, this research uses the multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model to analyze the primary crude oil benchmarks. Despite several GARCH model examinations, the crude oil benchmark remains elusive. Many theories attempted to explain why trends or moving averages may account for returns but random volatility cannot. However, historical returns and conditional variables stabilize volatility in GARCH models.

GARCH models can have an excessive number of parameters, Elder and Serletis (2011) stated that the optimal model for calculating multivariate GARCH and forecasting future fluctuations remains unknown as oil price affects the cost of several raw commodities. Predicting future prices to help investors make smart choices is as important as assessing risk. Production changes dramatically when oil prices unexpectedly rise or fall, according to Serletis and Elder (2011). It is the objective of investors to maximize profits while limiting risk and when it comes to estimating and forecasting, VAR and GARCH models work well (Ejukwa & Nanaka, 2024).

Several authors have modelled crude oil prices, Wiri and Tuaneh (2019) modelled crude oil price using ARIMA, Pre-Intervention and Post-Intervention model, Tuaneh and Wiri (2018), modelled crude oil price using the unrestricted VAR model, Tuaneh and Wiri (2021) modelled inflation rates and crude oil price using the Markov-Switching VAR. On the other hand, Tuaneh and Deebom (2019), used multivariate GARCH to model exchange rate and Nigerian deposit money market.

The multivariate GARCH model and variance prediction utilizing the same variables have only been used in a few research. Accordingly, the spillover volatility of the world's largest crude oil markets is notoriously hard to model. A multivariate GARCH model and prediction algorithms for many risk models accounting for various distributions like the model of Baba, Engle, Kraft, Kroner (BEKK) allow us to do this. Kanchan et al. (2017) examined the effect of volatility spread on spot and futures prices of black pepper using the multivariate GARCH model. Using VEC-BEKK, and Dynamic Conditional Correlation (DCC) model to evaluate the transmission of price signals from the spot and futures markets for black pepper, as well as the influence of volatility on other markets. Results for this analysis are based on those prices that appeared on the NCDEX in India between 2007 and 2013. The spot and futures markets for black pepper showed bidirectional volatility spillover, according to the researchers. The conditional relationship between spot and futures is dynamic and fluctuates over time, which caused uncertainty to extend to other markets. Additionally, the correlation between spot and futures prices may vary with the passage of time.

Using interest and currency rates from Nigerian commercial banks, Deebom et al. (2020) analyze diagonal MGARCH models using conditional variance-covariance. The analysis used the following variables: the annual exchange rate (EXR), the naira to USD exchange rates, the commercial bank time deposit interest rates (CBIRTD1, CBIRTD3, CBIRTD6, and CBIRTD12), and the monthly exchange rate. The data in question was collected during the months of 1991 and

2019. They used the Multivariate Diagonal VECH (DVECH) and BEKK-GARCH models to create the conditional variance-covariance matrix for five sets of time series data. Diagonal multivariate VECH GARCH analysis reveals that the covariances of these components do not cluster across time. The findings also showed that there was a positive conditional correlation between changes in the exchange rate and the interest rates on time deposits held by commercial banks.

Huang et al. (2010) employs the BEKK and DCC-GARCH multivariate GARCH models to ascertain the zero-coupon bond volatility. This research compares and analyzes the model selection criteria of many popular multivariate GARCH models. Financial data was retrieved from the website of the European Central Bank and used to fit the BEKK and DCC Multivariate GARCH Models independently. By comparing the findings, the best multivariate model was identified. We used a diagnostic test to check how well the data and model matched together.

The multivariate DCC-GARCH model was studied by Orskaug (2009) using different error distributions. As a multivariate GARCH model, Dynamic Conditional Correlation (DCC) is used. Predicting and ranking time series volatility is within the capabilities of GARCH models. A wide variety of variables are included in GARCH models. When creating many GARCH models, it is essential to simplify without sacrificing flexibility. It is essential that the conditional correlation matrix be positive definite as well. The DCC-GARCH model tweaks the association matrix to outdo the CCC-GARCH model. The DCC-GARCH model reduces processing time since the amount of correlated series has no effect on the number of parameters to estimate. This paves the way for the estimation of large association matrices. In the premise of most of these reviews, the concept of crude oil Average benchmark in the context of the BEKK model have little or no reference in the econometric world as nothing substantial have been credited to it. Besides, the variance between the correlation of raw price in Average, Brent, Dubai and West Texas Intermediate crude oil benchmark and its return when computed using Constant Conditional Correlation (CCC) model is still a grey area.

Statement of the Problem

This study examines the main crude oil markets through the lens of the multivariate GARCH model in order to ascertain averages, evolving correlations, and volatility transmission. Contradictory results have been produced by several GARCH model examinations of the crude oil market. Many hypotheses seek to clarify the discrepancy between the ability of random volatility to explain returns and the ability of trends or moving averages to do so. Two variables show the unequal effects when two or more potentially incorrect financial time series change at the same time. To stabilize volatility, the GARCH model employs conditional variables and historical returns. Optimal models for multivariate GARCH calculations and future fluctuation predictions do not yet exist, according to Serletis & Elder (2011). Many essential goods are influenced by oil prices. As important as risk assessment is for investors to make informed decisions, price forecasting is as crucial. An abrupt change in output happens if oil prices abruptly rise or fall, say Serletis and Elder (2011). Maximizing earnings while minimizing risk should be an investor's purpose. Estimating and predicting VaR is a good fit for GARCH models.

Aim of the Study

This study utilizes the multivariate GARCH Model to model the most significant crude oil markets' time-varying covariance, and spillover volatility.

Objectives of the Study

The objectives of this research include to;

i. ascertain the relationship between market volatility and crude oil price volatility by examining the spillovers between returns on Average, Brent, Dubai, and WTI crude oil prices;

ii. examine the linkages in the crude oil markets and how long they last; and

iii. investigate the disparity between correlation of the expected returns of the Average, Brent, Dubai, and West Texas intermediate raw price and it's return as computed using the constant conditional correlation of the DCC-GARCH.

Significance of the Study

The results of this research could be useful for students, entrepreneurs, professors, and others in positions of power. As they consider new laws and regulations, lawmakers may use this study as a resource for a deeper grasp of the market's inner workings. Additionally, market players could gain from better trading rules and more educated decision-making if they depend on trustworthy analysis of crude oil price changes. The main goal of this work is to add to what is already known about using MGARCH models to mimic the ebb and flow of crude oil market prices. Academics and students will find it beneficial as a complement to the current material.

Scope of the Study

To determine the correlation over time, and the spillover volatility, this study applies the multivariate GARCH Model to the world's biggest crude oil markets. Currently, we are looking at the BEKK GARCH and Constant Conditional Correlation model (CCC). The major emphasis of the study is for the period January, 1982 to April, 2023. One standard used to measure performance in the crude oil market is the benchmark crude, often called a marking crude. The study mainly considers four metrics: Average crude, Brent blend, Dubai crude, and WTI oil prices.

Materials and Method

Source of Data

The study data was sourced from the Central Bank of Nigeria (CBN) main website, www.cbn.gov.ng. The price per barrel ($/bbl) of crude oil from Brent (COB), Dubai (COD), and WTI (COW) is one of the components. From January 1982 to April 2023, the data collection covers the whole period with 1984 pieces of information.

Software use for Data Analysis

Econometric View (EView) 13 was used for this dataset. When it comes to data organization, visualization and analysis, EView is an all-inclusive statistical application. Researchers in econometrics, politics, economics, and biology may all benefit from its application in analyzing data trends and patterns.

Preliminary Analysis

For the logarithmic return and volatility, the data is fitted with a conditionally compound monthly return that is determined by the price of crude oil.

$$RCOA = Log\left(\frac{COA_t}{COA_{t-1}}\right) X \frac{100}{1} \quad (2.1)$$

$$RCOB = Log\left(\frac{COB_t}{COB_{t-1}}\right) X \frac{100}{1} \quad (2.2)$$

$$RCOD = Log\left(\frac{COD_t}{COD_{t-1}}\right) X \frac{100}{1} \quad (2.3)$$

$$RCOWTI = Log\left(\frac{COWTI_t}{COWTI_{t-1}}\right) X \frac{100}{1} \quad (2.4)$$

Each of the four types of returns on crude oil prices at time t are represented by equation 3.1, 3.2, 3.3 and 3.4. The transformation of price returns eliminates outliers and gets the variables to be stationary and dynamic.

Time Plot

Time maps display the information. It provides the first visual indicators of data series' trend, unit root, and volatility clustering.

Descriptive Statistics

The sanity test makes use of Jarque-Bera statistics. Chinyere *et al*. (2015) found that the Jarque-Bera test can determine whether data follows a normal distribution by looking at skewness and kurtosis. In 1980, Jarque and Bera reported this experimental result as;

$$X^2_{\underset{-}{\sim}} \quad \frac{N}{6}\left[S^2 + \frac{(K-3)^2}{4}\right] \quad (2.5)$$

A macroeconomic variable's magnitude is denoted by N, kurtosis by K, and skewness by S. This is the test statistic for a normal distribution when the null hypothesis is accepted.

Unit Root Test for Stationarity

In studies using random variables, this test is used to assess the degree of series integration. The unit root test is performed using the Augmented Dickey-Fuller (ADF) and the Phillip Perron Test (PPT). Assuming a series moves randomly is the premise of the unit root test.

$Y_t = \varphi_1 y_{t-1} + \varepsilon_t$ Random walk (2.6)

$Y_t = \varphi_0 + \varphi_1 y_{t-1} + \varepsilon_t$ Random walk with drift (2.7)

$Y_t = \varphi_0 + \varphi_1 y_{t-1} + \varphi_2 t + \varepsilon_t$ Random walk with drift and trend (2.8)

ARCH Effect

This test is used to evaluate whether the results produced from the model (3.6) do not meet the homoskedasticity assumption. The regression is then reported as follows;

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t+}^2 +,...,+\alpha\, p \varepsilon_{t-p}^2 + \mu_t \quad (2.9)$$

Where;

$\alpha_1,...,\alpha_p$ are the regression coefficients and α_0 is the intercept that is used to determine the null hypothesis?

H0: $\alpha_1 = \alpha_2 =, \ldots, = \alpha_p = o,$

The null on the assumption that there is no ARCH impact in the residuals.

H0: $\alpha_1 \neq \alpha_2 \neq ,...,\neq \alpha_p \neq o$

The alternative hypothesis that there is ARCH impact in the residuals.

Correlation Analysis

The correlation coefficient is the result of comparing the two returns on price using linear regression.

$$\rho_{x,y} = \frac{\operatorname{cov}(r_x, r_y)}{\sigma_x, \sigma_y} \tag{2.10}$$

The Multivariate GARCH Model Specification

We may generalize the univariate GARCH model to the multivariate model to get a better understanding of asset return relationships and financial econometrics in general. Theoretically, price changes in various assets and markets should follow regular patterns. Expanding univariate models into multivariate ones can improve your decision-making tools for value-at-risk (VaR) forecasting, hedging, portfolio selection, and asset pricing. This study analysis the Baba-Engle-Kraft-Krooner (BEKK) GARCH model, and the Constant Conditional Correlation (GARCH) model.

BEKK-GARCH Model

According to Engle and Kroner (1995), the word BEKK was named after Baba-Engle-Kraft-Kroner. This was propounded basically to improve on the VECH-GARCH model, the covariance matrix is a positive definite matrix, the parameters are easier to estimate, measuring the correlation and reflect the direction of spillover effects. The BEKK model by (Engle and Kroner, 1995) in matrix form is given as:

$$H_t = W^1 W + A^1 H_{t-1} A + B^1 \Xi_{t-1} \Xi_{t-1}^1 B \tag{2.11}$$

Where W is the upper triangular matrix of parameter, A and B an NxN parameter matrix, Ξ_{t-1} is the disturbance vector at time t-1.

Constant Conditional Correlation (CCC) Model

According to Hansen *et al.* (2012), the constant conditional correlation (CCC) model was developed by Bollerslev in 1990 to model the correlation coefficient matrix, but the coefficients are constant, describing univariate fluctuation characteristics and negatively capturing the dynamic correlation between sequences.

Let (η_t) be a sequence of iid variables with distribution η. A process (ϵ_t) is called CCC-GARCH(p,q) if it satisfies

$$\epsilon_t = H_t^{1/2} \eta_t$$
$$H_t = D_t R D_t \tag{2.12}$$
$$h_t = \omega + \sum_{i=1}^{q} A\, i \epsilon_{t-1} + \sum_{j=1}^{p} B_j h_{i-j}$$

where R is a correlation matrix, ω is an mx_1 vector with positive coefficients, A_i and B_j are mxm matrices with nonnegative coefficients, D_t is a diagonal matrix of conditional variance, $H_t^{1/2}$ is the cholesky factor of the time-varying conditional covariance matrix H_t, Dt is a diagonal matrix of conditional variance and η_t is an mx1 vector of normal, independent and identically distributed innovations.

The advantage of this specification is that a simple condition ensuring the positive definiteness of H_t is obtained through the positive coefficients for the matrices A_i and B_j and the choice for a positive definite matrix for *R*. However, this model is limited by its non-stability by aggregation and arbitrary nature of the assumption of constant conditional correlations.

Method of Estimating Parameters of Multivariate GARCH Model

The quasi-maximum likelihood (QML) method is often used for estimating the conditional covariance matrix of an MGARCH model. That is, if it is stated in the statement that θ is a parameter for a residual vector t with dimensions Nx1, and that the conditional covariance matrix of t, $H_t(\theta)$, is positive definite and NxN. By applying the log probability of a normal distribution to θ, the estimate can be optimize using the QML approach. $log\, L_T\,(\theta) = \frac{-N.T}{2} Log(2\Pi) - \frac{1}{2}\sum_{t=1}^{T} Log/H_t/ - \frac{1}{2}\sum_{t=1}^{T} \Xi_t^1 H_t^{-1} \Xi_t$ (2.13)

Post Estimation Technique

Conducting conditional heteroscedasticity tests is essential for ensuring that the chosen model provides the most accurate estimates. In this work, two tests were used to measure conditional heteroscedasticity. They are portmanteau analysis and the Q-Q plot

The Portmanteau test statistics is given as: $Q_k(m) = T^2 \sum_{i=1}^{m} \frac{1}{T-i} b_i(\hat{p}_o{}^{(a)-1} \otimes \hat{p}_o{}^{(a)-1}) b_i$

3 Results

Preliminary Test

As a component of our inquiry to gather information. Return plots for the four main benchmarks, time plots, descriptive statistics for raw and return on crude oil price benchmarks, correlation analysis, ARCH impact test and unit root tests were all executed.

Time Plot

Figures 4.1–4.4 provide the raw data on crude oil prices (naira/dollar) for WTI, Brent Blend, Dubai Crude, and the Average crude. Raw data is shown on the vertical axis and years on the horizontal axis. Figures 4.5, 4.6, 4.7 and 4.8 show plots of returns on benchmarks for crude oil prices. Looking at the price of crude oil in naira/dollar, WTI, Brent Blend, Dubai Crude, and the average all exhibit stagnant returns. According to the timeline, the series fluctuates at regular intervals around the origin.

Descriptive Statistics

A look at Table 4.2 reveals the raw and return prices for WTI, Brent Blend, Dubai Crude, and the Average benchmark for crude oil prices. The summary of the data characteristic is collated by testing whether the four crude oil benchmarks adhere to the normality assumption, the Jarque-Bera test is used in this assessment.

Correlation Analysis

Presented in table 4.2 is the results for Raw and Returned Crude Oil Prices Benchmark Correlation Analysis. Evaluating the coefficient of determination for the correlation between benchmarks for crude oil prices and price returns is paramount. Also, in table 4.4 is the results for the difference between the correlation of raw price benchmark and its return when estimated using Constant Conditional Correlation of the Diagonal Conditional Correlation.

Unit Root Test

This test is conducted to check for the presence of unit root and to determine the order of integration of the crude oil benchmarks. Table 4.3 compares stability in the context of stationarity and according to the findings, the series reaches a stationary point at the first difference.

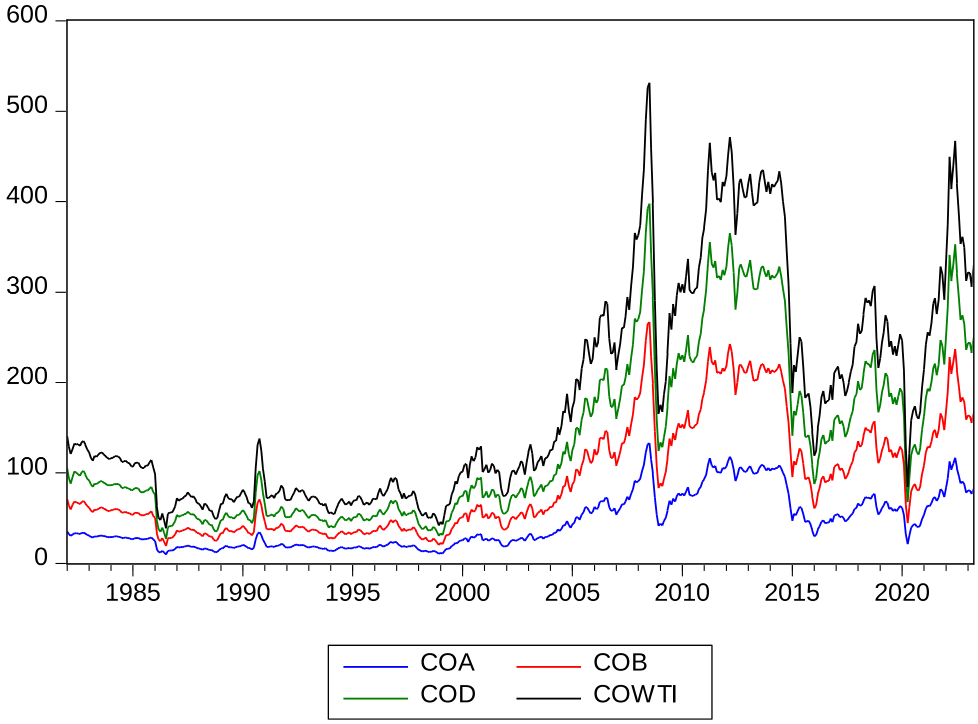

Figure 1: Time Plot on Crude Oil Price Average, Crude Oil Price Brent, Crude Oil Price Dubai and Crude Oil Price West Texas Intermediate from 1982, January to May, 2023.

Mean	44.885	46.0149	43.899	44.741	0.173	0.172	0.182	0.164
Median	30.700	30.975	28.950	31.730	0.902	0.565	0.866	0.801
Maximum	132.830	133.870	131.220	133.930	43.020	43.263	49.102	54.744
Minimum	9.620	9.450	7.850	11.310	-50.491	-51.143	-54.012	-59.262
Std. Dev.	30.395	31.741	31.162	28.463	9.163	9.3701	9.474	9.411
Skewness	0.890	0.9149	0.903	0.876	-0.677	-0.549	-0.707	-0.676
Kurtosis	2.603	2.657	2.612	2.648	8.313	6.859	9.274	11.028
Jarque-Bera	68.692	71.623	70.548	65.952	620.109	331.795	853.059	1367.020

Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum	22263.08	22823.37	21774.10	22191.62	85.554	85.278	90.243	81.112
Sum Sq. Dev.	457293.6	498712.3	480674.9	401026.6	41482.23	43372.71	44335.77	43753.33
Observations	496	496	496	496	495	495	495	495

Table 1: Descriptive Statistics on Raw and Return on Crude Oil Price Bench Mark

Table 2: Correlation Analysis between Raw and Returns on Crude Oil Price Benchmarks

Variables	COA	COB	COD	COWTI	RCOA	RCOB	RCOD	RCOWTI
COA	1.0000							
COB	0.9992 0.0000	1.0000						
COD	0.9986 0.0000	0.9988 0.0000	1.0000					
COWTI	0.9960 0.0000	0.9923 0.0000	0.9903 0.0000	1.0000				
RCOA	0.0664 0.1404	0.0656 0.1452	0.0602 0.1815	0.0736 0.1020	1.0000			
RCOB	0.0653 0.1468	0.0659 0.1433	0.0594 0.1870	0.0707 0.1162	0.9864 0.0000	1.0000		
RCOD	0.0673 0.1348	0.0663 0.1407	0.0631 0.1607	0.0725 0.1069	0.9792 0.0000	0.9643 0.0000	1.0000	
RCOWTI	0.0613 0.1733	0.0593 0.1881	0.0533 0.2364	0.0719 0.1101	0.9672 0.0000	0.9268 0.0000	0.9100 0.0000	1.0000

Table 3: Results for ARCH Impact Test

Crude oil Price Benchmark series	**Residuals Heteroscedasticity Test (statistics)**	**P-Value**
Crude oil Price Average	10701.210	(0.0000)
Crude oil Price in Brent Blend	22431.320	(0.0000)

Crude oil Price in Dubai	8643.737	(0.0000)
Crude oil Price in West Texas Intermediate (WTI),	7906.609	(0.0000)

Table 4: Unit Root Test on Raw and Return on Crude Oil Price Benchmarks

Variable	t-Statistic	P-Value	Remarks
COA	-2.220	0.199	
D(COA)	-14.976	0.000	**1(1)**
COB	-2.172	0.217	
D(COB)	-15.415	0.000	**1(1)**
COD	-2.171	0.217	
D(COD)	-14.637	0.000	**1(1)**
COWTI	-2.367	0.152	
D(COWTI)	-15.396	0.000	**1(1)**

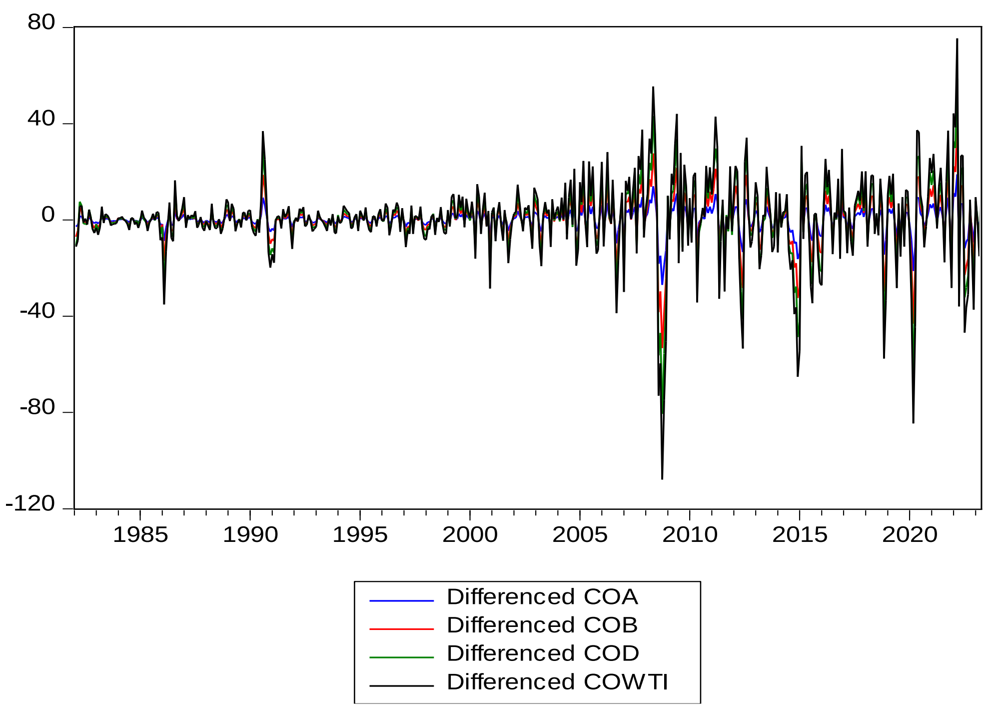

Figure 2: Time Plot on the Difference Crude Oil Price Average, Crude Oil Price Brent, Crude Oil Price Dubai and Crude Oil Price West Texas Intermediate from 1982, January to May, 2023.

Figure 3: Results for Estimates of Diagonal Baba-Engle-Kraft-Kroner(BEKK) Model

$$M = \begin{bmatrix} 8.161 & & & \\ & 8.161 & & \\ & & 8.161 & \\ & & & 8.161 \end{bmatrix}, \quad A1 = \begin{bmatrix} 0.201 & & & \\ & 0.187 & & \\ & & 0.215 & \\ & & & 0.194 \end{bmatrix},$$

$$B1 = \begin{bmatrix} 0.847 & & & \\ & 0.855 & & \\ & & 0.838 & \\ & & & 0.851 \end{bmatrix}$$

Variance Equation

$$\sigma^2_{1,t} = M + A1(i,j)^2\varepsilon^2_{i,t-1} + B1(i,j)^2\sigma^2_{i,t-1}$$

Where $M > 0$ and M is a scalar, $A1(i,j)$ *and* $B1(i,j)$ are diagonal matrix.

$$\sigma^2_{1,t} = \begin{bmatrix} 8.161 \\ 8.161 \\ 8.161 \\ 8.161 \end{bmatrix} + \begin{bmatrix} 0.201 \\ 0.187 \\ 0.215 \\ 0.194 \end{bmatrix} \varepsilon^2_{i,t-1} + \begin{bmatrix} 0.847 \\ 0.855 \\ 0.838 \\ 0.851 \end{bmatrix} \sigma^2_{i,t-1}$$

Covariance Equation

$$\rho_{1,2,t} = 8.161 + 0.194\varepsilon_{1,t-1} * \varepsilon_{2,t-1} + 0.851\rho_{1,2,t-1}$$
$$\rho_{1,3,t} = 8.161 + 0,208\varepsilon_{1,t-1} * \varepsilon_{3,t-1} + 0.842\rho_{1,3,t-1}$$
$$\rho_{1,4,t} = 8.161 + 0.204\varepsilon_{1,t-1} * \varepsilon_{4,t-1} + 0.846\rho_{1,3,t-1}$$
$$\rho_{2,3,t} = 8.161 + 0.201\varepsilon_{2,t-1} * \varepsilon_{3,t-1} + 0.846\rho_{1,3,t-1}$$
$$\rho_{2,4,t} = 8.161 + 0.196\varepsilon_{2,t-1} * \varepsilon_{4,t-1} + 0.850\rho_{2,4,t-1}$$
$$\rho_{3,4,t} = 8.161 + 0.211\varepsilon_{3,t-1} * \varepsilon_{4,t-1} + 0.841\rho_{3,4,t-1}$$

Table 5: Results of Difference Between Correlation and Constant Conditional Correlation of the DCC-GARCH

Correlation	**0.989**	**0.979**	**0.970**	
Heterogeneous Variations	0.986	0.979	0.967	
Correlation		0.935	0.936	
Heterogeneous Variations		0.964	0.927	
Correlation				0.915
Heterogeneous Variations				0.910

Table 6: Estimation Results for Portmanteau Tests

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	Df
1	22.93649	0.1154	23.05721	0.1122	16
2	46.25668	0.0494	46.62417	0.0458	32
3	59.27973	0.1274	59.85504	0.1171	48
4	91.70694	0.0132	92.97588	0.0105	64
5	100.8715	0.0574	102.3868	0.0466	80
6	113.1738	0.1113	115.0881	0.0896	96
7	122.4939	0.2343	124.7627	0.1931	112
8	155.4524	0.0497	159.1621	0.0322	128
9	172.4273	0.0532	176.9764	0.0321	144
10	192.1867	0.0420	197.8274	0.0225	160
11	204.9934	0.0664	211.4168	0.0352	176
12	223.8200	0.0576	231.5055	0.0271	192

Source: Researcher's Calculation using Eviews Version 10

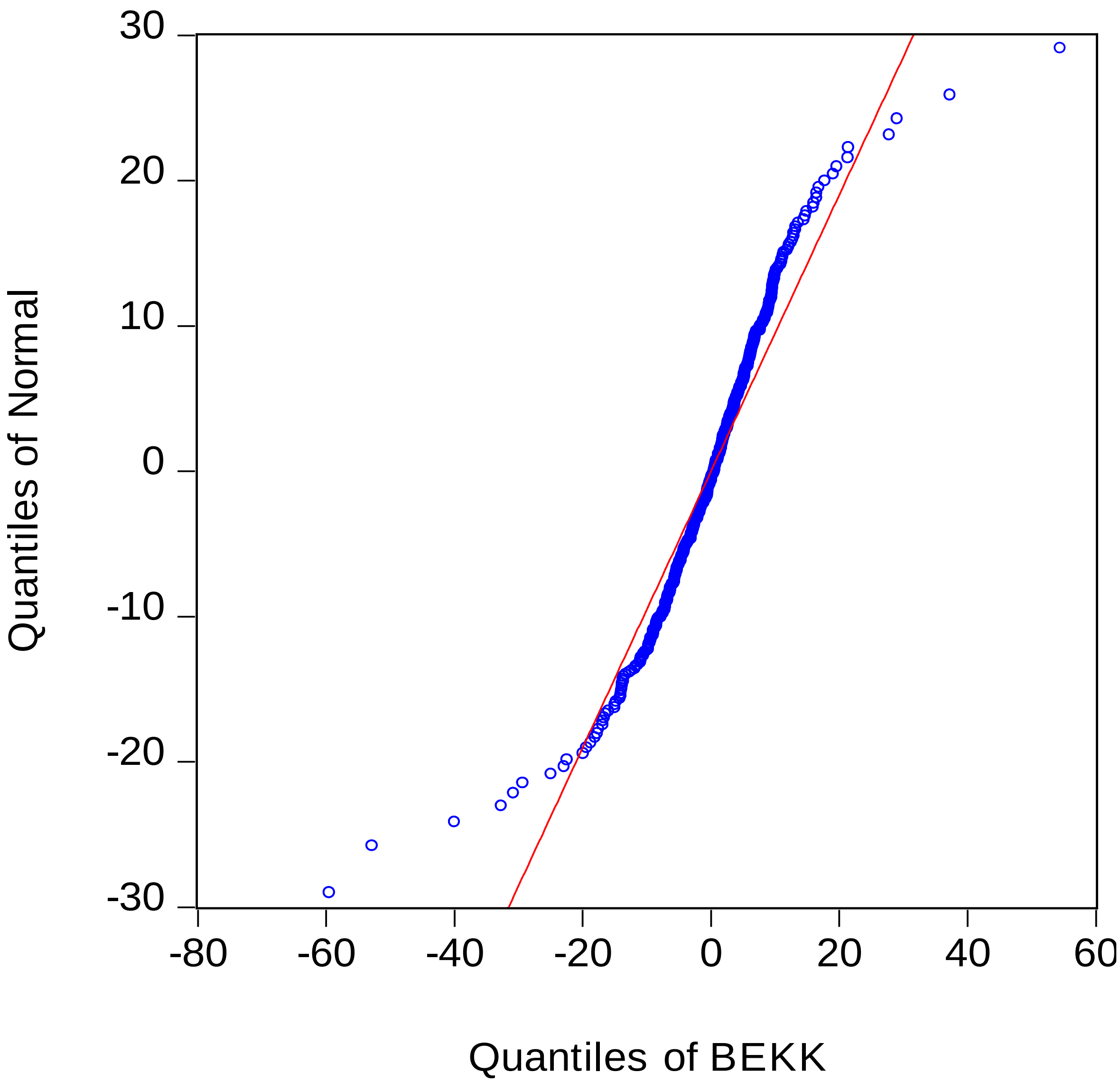

Figure 4: QQ Plot on the BEKK Model to Determine the Adequacy of the Model.

5 Discussion of Results

Time Plot

The series behaviour was shown by Figures 1 which provided the raw crude oil price data (naira/Dollar). All four major price indexes agree that crude oil prices peaked in January 2006 and began a precipitous fall in June 2008. Using the de-trending method, we can see how the price of crude oil has changed in relation to our study. There is a unifying trend across all price indexes when viewed against time. Clustering volatility is shown graphically in Figures 2 which depict stationary crude oil price fluctuations near the origin.

Raw and Returns on the Nigerian Crude Oil Price Descriptive Statistics

Table 1 displays the raw and returned crude oil prices from Nigeria. The Jacque-Bera test was used to see whether the returns on the price benchmark were normal. Positive skewness is evident in the following COA(0.890), COB(0.99149), COD(0.903), and COWTI (0.876). Despite the right-leaning nature of the raw data points, the average return series for Brent Blend, Dubai Crude, West Texas Intermediate, and Crude Oil Average are left-leaning. Information about the tail is given by kurtosis. All four raw series have kurtosis values below 3: COA(2.603%), COB(2.656%), COD(2.612%), and COWTI (2.648). In contrast, RCOA(8.313), RCOB(6.859), RCOD(9.274), and RCOWTI (11.328) with values higher than 3 indicate a leptokurtic distribution.

Correlation Analysis

Using correlation analysis, it may try to make sense of the relationships between the variables in a single development. At the 5% level of significance, any coefficient of determination for these variables is remarkable as shown in table 2. The returns on average crude oil prices (RCOA) are not significantly correlated with any of the other variables (COA:0.0664, COB:0.0656, COD:0.0602, COWTI:0.0736).

Reversely, the correlation between average (RCOA) and Brent Blend (RCOB) returns is strong and positive; the coefficient of determination is 0.9864 at the 5% level of significance.

West Texas Intermediate (RCOWTI), average (RCOA), Brent Blend (RCOB), and Dubai Crude (RCOD) all have positive and statistically significant correlations to their returns. The coefficients of determination are 0.9656, 0.9268, and 0.9100 at the 5% level of significance.

ARCH Effect

Table 6 displays the results of the test for heteroskedasticity, which is also called the ARCH effect. Any GARCH model worth its salt must account for heteroscedasticity. All the four crude oil benchmarks are significant at 5% level of significance. The null hypothesis is upheld, that there is presence of ARCH effect in all the variables.

Diagonal Baba-Engle-Kraft-Kroner (BEKK) Model Estimates

In the diagonal BEKK model, the volatility spillers and volatility series allow one to study the conditional variance, covariance, and correlation. As can be observed from their logarithmic returns, the co-movement of the variables is dynamic and time-varying. A1(1,1), A1(2,2), A1(3,3), and A1(4,4) in figure 3 are matrices with diagonal parameters that quantify the conditional variance's sensitivity to previous market volatility and shocks. B1(1,1), B1(2,2), B1(3,3), and B1(4,4) are matrices with the same information. The symmetric BEKK-GARCH (1,1) model demonstrated that all parameters were statistically significant at the 5% level. The shock variance equation's coefficients A1(1,1) (0.201), A1(2,2) (0.187), A1(3,3) (0.215), and A1(4,4) (0.194) are shown diagonally. Crude oil prices felt the most disturbance in Dubai, then average crude, followed by West Texas intermediate and then Brent benchmark. The following shows how previous shocks affected conditional variance. Similarly, the delayed shocks now positively impact the crude oil market's conditional volatility. Additionally, there are volatility spillovers with B1(1,1) = 0.847, B1(2,2) = 0.855, B1(3,3) = 0.838, and B1(4,4) = 0.851. According to Matrix B1, the shock duration is greatest for Brent, followed by WTI, the Average crude oil price, and Dubai. Due to the low shock persistence and stability in Dubai, it proves to be the gold standard for measuring crude oil prices. This shows the effect of own lagged volatility on the current

conditional volatility of the crude oil price on market return benchmark. In all markets, the past volatility coefficient is larger than the past shock coefficient, indicating previous volatilities rather than previous shocks are important for forecasting future volatilities.
According to the data shown here, benchmark market returns are far less influential on the development and conditional volatility of crude oil market prices than shocks and delayed volatility. From what we can see, GARCH appears to have a major impact whereas ARCH has a little one as shown by their coefficients in figure 9. Someone clearly cares more about out-of-date knowledge than current happenings if they are engaged in the sale or investing of crude oil. Past values and interactions of variables determine the model's expected variance coefficients. The variance is determined only by the squared residuals from before, whereas the interaction between the past squared residuals and the diagonal GARCH components determines the covariance. Prior conditional covariance terms (βij) and prior error terms (αij) have substantial and positive cross-products for each return in the crude oil market. Volatility spillover affects certain markets as Crude oil from Dubai is immediately impacted by shocks in the Brent crude oil market, also the West Texas Intermediate spillover to Dubai. Shock to Brent crude oil had a discernible effect on the average market for crude oil. It seems that the volatility in certain crude oil markets is having a two-way effect. This research confirms the results of Kanchan et al. (2017) by examining the effects of volatility spillover on black pepper spot and future prices using a multivariate GARCH model. Table 5 displays the results of the diagonal conditional correlation (DCC) and constant conditional correlation (CCC). A variation of 0.003 was found in the correlation between Brent's raw price and Average compared to its returns, according to the DCC-GARCH's constant conditional correlation. The raw price correlation in Dubai is consistent with the Average and anticipated returns of the DCC-GARCH constant conditional correlation. Also, the difference between correlation and constant conditional correlation of the DCC-GARCH raw price in West Texas Intermediate and Average, and it returns is 0.003. There is a 0.029 difference in the raw Brent and Dubai prices caused by the DCC-GARCH compared to the constant conditional correlation. Also, the DCC-GARCH correlation and constant conditional correlation are different from the returns of Brent and Dubai raw price by 0.009. Finally, the variation between the correlation and constant conditional correlation of the DCC-GARCH raw price returns for Dubai and West Texas Intermediate is 0.005.

Post-estimation findings
Diagnostic tests on the model was conducted to determine its sufficiency. Table 6 shows the residual Portmanteau test findings for autocorrelations up to lag h. There are no autocorrelations detected by the portmanteau test estimate up to lag h with most of the p-values less than 0.05. Using a QQ Plot as shown in Figure 4 to support this argument, the dots on the p-p plot are closely distributed along the fitted line which gives credence to the validity of the BEKK model.

5. Conclusion
There is a unifying trend across all price indexes when viewed against time and clustering volatility is established which depict stationary crude oil price fluctuations near the origin.

Furthermore, the diagonal BEKK MGARCH model indicates that conditional volatility and prior squared errors have a significant impact on the conditional variances of the four key returns for crude oil price benchmarks. Shocks don't stick around for long, and people in this market tend to focus on old news instead of fresh information. Most importantly, the conditional covariances were affected by the cross-products of the previous error and covariance terms. As a result, the study concludes that the volatility of the crude oil markets examined exhibit strong linkages and bilateral volatility transmission from one market to the other. It was also established that there is no disparity between correlation of the expected returns of the Average, Brent, Dubai, and West Texas intermediate raw price and it's return as computed using the constant conditional correlation of the DCC-GARCH. The diagonal BEKK-GARCH model is sufficient as confirmed by the portmanteau test and the QQ plot test.

Recommendations

i. The crude oil market needs the central bank to prioritize price stabilization so that "petro-aggression" may be avoided.
ii. In order to reduce price volatility and enhance the early warning system for crude oil price crises, the government should enhance its system for recognizing volatility spillover effects between Average, Brent, Dubai, and WTI Crude Oil returns.
iii. To better manage risk and rebalance their portfolios, marketers and investors should benefit from a better understanding of the link between crude oil prices
iv. Since crude oil market factors such as mean, time-varying covariance, and spillover volatility are interdependent, it is necessary to use a multivariate GARCH Model to assess the advantages of this dependency.

Contribution to Knowledge

This work has contributed new and important information in the following ways:

i. In the diagonal BEKK MGARCH model, volatility and previous squared errors have a significant impact on the conditional variances of the four main crude oil price benchmark returns. Marketers and investors in the crude oil industry pay greater attention to breaking news items than to shock persistence.

Indicating that the studied crude oil markets went through significant volatility co-movements, the conditional covariances were significantly affected by prior error terms and prior covariance terms.

ii. During the course of the research, there was a robust correlation between these variables, which may indicate a rise in volatility.

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